Design Project – Optimum Material Selection for a Telescope Mirror
ME 280 – Introduction to Materials Science
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I. EXECUTIVE SUMMARY

This design project[1] is designated to explore the optimal material for a mirror plate in terms of different criteria such as economic, dynamic, thermal, mechanical, and safety criteria. Based on computational results and conducted resources, the optimal material is chosen to be Reaction-bonded Silicon nitride (RBSN). The RBSN allows the mirror plate to be created with the (minimum) mass per unit surface area of 1085.31 $[kg/m^2]$ and the (minimum) thickness of 401.96 $[mm]$ without being fractured under temperature change or its gravitational force.

II. PROBLEM DESCRIPTION

The purpose of this project is to find the optimal material for a telescope mirror. A variety of glasses, crystals, ceramics, and metals will be studied. 15 different materials and their properties will be taken into consideration. The criterions that will be studied to recommend a material that is sufficient include economic, thermal, dynamic, mechanical, safety, and environmental criterions.

III. DESIGN CONSIDERATIONS (TECHNICAL, SAFETY, ECONOMIC, ENVIRONMENTAL)

In choosing the optimal material for the mirror plate, three technical criteria are taken into consideration. Dynamically, the mirror plate's minimum natural frequency must be 500 Hz. In terms of thermal criterion, suppose that the mirror plate experiences a transition in temperature, from 30°C (high room temperature) to -10°C (night-air temperature). Hence, it must not fracture due to thermal stress. The last technical criterion is mechanical. If the mirror plate is simply supported, because the thickness is too thin, it must be guaranteed that the plate does not fracture under its own weight. To make sure the thermal and mechanical stresses are verified and to avoid hazardous accidents, the factor of safety is given as 2.0.

Considering the economical aspect of choosing the ideal material of the mirror, the lighter the mirror plate, the more optimal it is economically. The mass of the mirrors per material were calculated through combining both the Dynamic and Mechanical equations below. To satisfy all criteria, in comparing the Dynamic and Mechanical masses, the larger value is decided to be the minimum satisfies mass. Therefore, the lighter the mass, the more economical the material is.

The production of most materials affects the environment through atmospheric emission of gasses in the manufacturing and distribution processes. For example, the decomposition of raw material through melting with combustion of oil and natural gas lead to CO_2 emissions. Moreover, mirrors are generally not recyclable. However, many mirrors used today are made of natural materials (ex. glass) that would last for a long time.

IV. ASSUMPTIONS, PROPERTY SELECTION, DISCUSSION

Usually, telescope mirrors are made by adding a thin metallic coating layer (the mirror) on a supporting plate. The plate is mounted by simple supports around the edges of the mirror to allow free expansion. The mirror must be circular with a diameter of 2 $[m]$ and a uniform thickness. Moreover, during the thermal cycling of the external sides, the temperature varies between 30°C to -10°C. The lowest frequency for the simply supported plate is 500 $[Hz]$.

The 12 different materials that are studied in this project, and their properties are listed below in Table 1.1

Table 1.1: Material properties of 12 given candidate materials

	Material	Density ρ $[kg/m^3]$	CTE α $[10^{-6}/K^{-1}]$	Poisson's ratio v	Young's modulus E $[GPa]$	$\sigma_{fracture}$ $[MPa]$	Thermal conductivity k $[W/m°C]$	Heat capacity c $[J/kg°C]$
1	**Borosilicate glass, BK7**	2510	7.1	0.23	82	50	1.1	750
2	**Fused silica, SiO_2**	2190	0.5	0.17	73	50	1.1	750
3	**Single crystal quartz, SiO_2**	2640	11	0.25	73	50	8	750
4	**Silicon, Si**	2330	2.5	0.24	160	120	150	750
5	**Beryllium, Be I-70**	1850	11.3	0.25	290	240	220	1900
6	**Aluminum, Al 6061**	2700	22.5	0.33	68	280	170	900
7	**Aluminum, Al 7075**	2740	26	0.33	71	500	120	1000
8	**Stainless Steel 304**	7830	17.3	0.33	193	515	16	500
9	**Ti Alloy, Ti-6Al-4V**	4430	8.6	0.33	114	900	6.7	610
10	**Mg Alloy, Mg AZ91D**	1810	26	0.33	45	200	72	1050
11	**Ceramic, SiC**	2950	2.4	0.18	360	300	170	670
12	**Ceramic, ZrO_2**	6000	9.6	0.25	205	1150	2.5	480

Table 1.2: Characteristics of mirror plate using 12 given candidate materials

	Material	Thermal test*	Minimum dynamic mass $[kg]$	Minimum dynamic thickness $[mm]$	Minimum mechanical mass $[kg]$	Minimum mechanical thickness $[mm]$
1	**Borosilicate glass, BK7**	0	5870.39	744.46	9.41	1.19
2	**Fused silica, SiO_2**	1	5134.55	746.29	7.03	1.02
3	**Single crystal quartz, SiO_2**	0	6677.22	805.09	10.47	1.26
4	**Silicon, Si**	1	3749.36	512.21	3.39	0.46
5	**Beryllium, Be I-70**	0	1965.21	338.13	1.07	0.18
6	**Aluminum, Al 6061**	1	6976.22	822.44	2	0.24
7	**Aluminum, Al 7075**	1	6979.52	810.82	1.16	0.13
8	**Stainless Steel 304**	1	20449.97	831.34	9.16	0.37
9	**Ti Alloy, Ti-6Al-4V**	1	11323.53	813.63	1.68	0.12
10	**Mg Alloy, Mg AZ91D**	1	4706.96	827.77	1.26	0.22
11	**Ceramic, SiC**	1	3608.24	389.33	2.13	0.23
12	**Ceramic, ZrO_2**	1	13652.15	724.27	2.35	0.12

** Thermal test indicates 0 if the materials fail thermally, and 1 if the materials pass the thermal test*

To consider **thermal criterion**, the maximum thermal stress in Equation (1a) must not exceed the allowable fracture stress as seen in Equation (1b).

$$\sigma_{\max thermal} = \frac{\alpha E \Delta T}{1 - v} \qquad (1a)$$

$$\sigma_{\max thermal} \leq \frac{\sigma_{fracture}}{N} \tag{1b}$$

$$N \cdot \Delta T \leq \frac{\sigma_{fracture}(1-v)}{\alpha E} \tag{2}$$

Combining Equations (1a) and (1b), Equation (2) is obtained, where $N = 2$ is the given factor of safety, $\Delta T = 30 - (-10) = 40$°C is the change in temperature, and ($\sigma_{fracture}$, α, E, v) are the given properties of each material (see Table 1.1). If Equation (2) is satisfied, the material passes the thermal test. This is calculated via the Matlab code (see Appendix). If the "thermal" output is "1" then the material passes the thermal test, and if the output is "0" then the material fails.

In checking the **dynamic criterion**, the natural frequency of the mirror plate must be at least 500 $[Hz]$ to prevent excessive shaking. Equation (3) below is constructed to satisfy that criterion.

$$f = \frac{1}{2\pi} \cdot \frac{2.489}{R^2\sqrt{\frac{12(1-v^2)\rho}{E \cdot t^2}}} \geq f_{min} = 500\ Hz \tag{3}$$

Where $R = 1[m]$ is the mirror's radius, $t\ [m]$ is the thickness, and (v, ρ, E) are the given properties of the materials. By rewriting Equation (3), Equations of thickness and mass (4 and 5 respectively) are obtained.

$$t_{dynamic} \geq 2\pi \cdot f_{min} \cdot \frac{R^2\sqrt{12(1-v^2)^{\rho}/_{E}}}{2.489} \tag{4}$$

$$m_{dynamic} = \pi R^2 \cdot t_{dynamic} \leq \frac{2\pi^2\sqrt{12}}{2.489} \cdot R^4 \cdot f_{min} \cdot \sqrt{1-v^2} \cdot \frac{\rho^{\frac{3}{2}}}{E^{\frac{1}{2}}} \tag{5}$$

To avoid the material fracturing on its own weight, the **mechanical criterion** is considered, where the maximum bending stress must not exceed the allowable fracture stress as in Equation (6)

$$\sigma_{\max mechanical} \leq \frac{\sigma_{fracture}}{N} \tag{6}$$

Through rewriting Equation (6), the Equations of thickness and the mass (7 and 8 respectively) are obtained as follows.

$$t_{mechanical} \geq \frac{3(3+v)}{8} \cdot \rho g \cdot R^2 \cdot \frac{N}{\sigma_{fracture}} \tag{7}$$

$$m_{mechanical} = \pi R^2 \cdot t_{mechancial} \geq \frac{3\pi}{8} \cdot g \cdot N \cdot R^4 \cdot \frac{\rho^2(3+v)}{\sigma_{fracture}} \tag{8}$$

Both Equations (5) and (8) give the minimum mass based on dynamic and mechanical criteria respectively. Therefore, to satisfy both criteria, the larger mass value is chosen to be the minimum mass of the mirror plate. From Table 1.2, all materials have dynamic mass higher than mechanical mass, so conclusion in choosing the mirror's mass should only be based on the **dynamic criterion**.

According to Table 1.2, among the given materials that passed the thermal test, the mirror plate made of Ceramic (SiC) has the lightest mass of 3608.24 $[kg]$. Following is Silicon (Si) mirror which has a mass of 3749.36 $[kg]$. Mirrors made of Mg alloy (Mg AZ91D) and Fused Silica (SiO_2) are considered the 3rd and 4th lightest ones. Hence, the 3 proposed materials should ideally have a high percentage of Si (SiC, SiO_2) or Mg. After testing all the materials provided in Wiley (2015)[2] that contain Si or Mg, the materials that pass the thermal test and give the lightest masses are: Silicon nitride (Reaction-bonded), Silicon carbide (Sintered), and Glass ceramic (Pyroceram).

Table 2.1: Material properties of 3 proposed materials

	Material	Density ρ $[kg/m^3]$	CTE α $[10^{-6}/K^{-1}]$	Poisson's ratio v	Young's modulus E $[GPa]$	$\sigma_{fracture}$ $[MPa]$	Thermal conductivity k $[W/m°C]$	Heat capacity c $[J/kg°C]$
1	**Silicon nitride, Reaction-bonded**	2700	3.1	0.22	304	297.5	10	870
2	**Silicon carbide, Sintered**	3200	4.1	0.16	345	308	71	590
3	**Glass-ceramic, Pyroceram**	2600	6.5	0.25	120	246.5	3.3	975

Table 2.2: Characteristics of mirror plate using 3 proposed materials

	Material	Thermal test	Minimum dynamic mass $[kg]$	Minimum dynamic thickness $[mm]$	Minimum mechanical mass $[kg]$	Minimum mechanical thickness $[mm]$
1	**Silicon nitride, Reaction-bonded**	1	3409.59	401.96	1.82	0.22
2	**Silicon carbide, Sintered**	1	4178.78	415.67	2.43	0.24
3	**Glass-ceramic, Pyroceram**	1	5090.03	623.16	2.06	0.25

Combining the top four materials from Table 1.2 which are Ceramic (SiC), Silicon (Si), Mg alloy (Mg AZ91D), and Fused Silica (SiO_2), with the three proposed materials in Table 2.2, the five materials that would give the five lightest masses of the mirror plate are indicated in Table 3, ranked from the lightest to the heaviest.

Table 3: Characteristics of mirror plate from the top 5* materials

**Top 5 indicates the 5 materials that give the lightest masses and pass all the criteria*

	Material	Thermal test	Minimum dynamic mass [kg]	Minimum dynamic thickness [mm]	Minimum mechanical mass [kg]	Minimum mechanical thickness [mm]
1	**Silicon nitride, Reaction-bonded**	1	3409.59	401.96	1.82	0.22
2	**Ceramic, SiC**	1	3608.24	389.33	2.13	0.23
3	**Silicon, Si**	1	3749.36	512.21	3.39	0.46
4	**Silicon carbide, Sintered**	1	4178.78	415.67	2.43	0.24
5	**Glass-ceramic, Pyroceram**	1	5090.03	623.16	2.06	0.25

V. DESIGN CONCLUSIONS

In determining the optimal material for the mirror plate, other than the technical, economic, thermal, dynamic, mechanical, and safety criteria, the environmental factors must also be considered. Five materials from Table 3 are compared in terms of advantages and disadvantages regarding toxicity emission, cost of materials, environmentally friendly to consumers, applications, etc.

Table 4: Advantages and Disadvantages of top 5* materials

**Top 5 indicates the 5 materials that give the lightest masses and pass all the criteria*

<table>
<tr><th></th><th>Material</th><th>Mass [kg]</th><th>Advantages</th><th>Disadvantages</th></tr>
<tr><td>1</td><td>Silicon nitride, Reaction-bonded</td><td>3409.59</td><td>- Can be produced in one single process stage
- Reduce fuel consumption and emission
- High melting point, avoid toxic fumes from emitting in normal conditions[3]</td><td>- Hard to achieve the complete bond reaction
- At decomposition temperature, toxic fumes of ammonia and ozone may emit
- Health hazards are not well studied yet[4]</td></tr>
<tr><td>2</td><td>Silicon, Si[5]</td><td>3749.36</td><td>- Low cost, least expensive infrared material
- Lightweight, high thermal and mechanical durability (popular in making mirrors)</td><td>- High absorption characteristics</td></tr>
<tr><td>3</td><td>Ceramic, SiC</td><td>3608.24</td><td rowspan="2">- Sizes range from 100 to 1000 mm
- Cover a range of wavelength from the extreme UV, X-ray radiation, to the sub-millimeter waves[6]</td><td rowspan="2">- Expensive
- Long machining time
- If contact, can irritate eyes and nose
- Frequent exposure can cause chronic disease of the lungs[7]</td></tr>
<tr><td>4</td><td>Silicon carbide, Sintered</td><td>4178.78</td></tr>
<tr><td>5</td><td>Glass-ceramic, Pyroceram[8]</td><td>5090.03</td><td>- Handles extreme heat, do not expand much due to low CTE
- Better than glass made of Si/SiO2: sustains higher heat and strength</td><td>- Easily fractures
- Relatively expensive</td></tr>
</table>

It is concluded that **Silicon nitride, Reaction-bonded (RBSN)** is the optimal material to design a mirror plate. The RBSN allows the mirror plate to be created with the (minimum) mass per unit surface area of 1085.31 $[kg/m^2]$ and the (minimum) thickness of 401.96 $[mm]$ without being fractured under temperature change or its gravitational force, at the same time maintaining the minimum natural frequency of 500 $[Hz]$. Therefore, RBSN satisfies all required criteria: dynamic, thermal, mechanical, economic, and safety.

Additionally, it is also proved to be the most suitable material considering the material cost, health hazards, and environmental effects. RBSN is made by nitriding Silicon or compact at 1450°C. It has a slightly higher density than other glass-ceramics due to porosity accompanied by dimensional change. This reaction contributes to the favorable economic advantage since there is usually no need for further expensive grinding other than the reaction itself. Like most glass-ceramic, RBSN has a low CTE which grants it an outstanding resistance to shock change of temperature and heat. The major advantages of RBSN are good optical properties and thermal properties. Potential disadvantages could be inferior mechanical properties due to the porosity. However, in this design project, the major mechanical criterion for design was its ability to hold its own weight which was well satisfied as shown above. In this case, this inferior property disadvantage can be neglected. In conclusion, these factors make **Silicon nitride, Reaction-bonded (RBSN)** the optimal material for this design project.

VI. ACKNOWLEDGEMENTS

We acknowledge the guidance and support from Professor Lambropoulos and the entire Teaching Assistant team in completing the design project.

VI. REFERENCES

[1] Design Project Manual, ME 280 – Introduction to Material Sciences, University of Rochester

[2] Wiley (2015), "Fundamentals of Materials Science and Engineering", Appendix B, p. 883

[3] "Silicon Nitride Properties and Applications", *AZO Materials*, https://bit.ly/Silicon-nitride-1

[4] "Silicon Nitride", *ESPI Metals*, https://bit.ly/Silicon-nitride-2

[5] *RMI*, http://rmico.com/optics-tutorial

[6] "Development of silicon carbide mirrors: the example of the ...", *SPIE.*, https://bit.ly/SiC-1

[7] New Jersey Department of Health and Senior Services, https://bit.ly/SiC-2

[8] "Glass-ceramic", *Wikipedia*, https://en.wikipedia.org/wiki/Glass-ceramic

VII. APPENDIX

```
%% ME 280 Design Problem
% Calculation of each criterion
% Zainab, Zheming, Linh

%% Load data

data = readtable('Materials'' Properties.xlsx');
mat_name = table2array(data(:,1));                 % Materials' names
p = table2array(data(:,2));                        % Density [kg/m^3]
CTE = table2array(data(:,3))*10^-6;                % Co-e thermal expansion [1/C]
v = table2array(data(:,4));                        % Poisson's ratio [-]
E = table2array(data(:,5))*10^9;                   % Young's modulus [Pa]
stress_fracture = table2array(data(:,6))*10^6;     % Fracture stress [Pa]

%% Given

g = 9.81;             % Gravitional acceleration [m/s^2]
R = 1;                % Radius of the plate [m]
fmin = 500;           % Minimum natural frequency [Hz]
N = 2;                % Factor of safety [-]
delta_T = 40;         % Change in temperature [Celcius]

%% Loop

[row,col] = size(mat_name);

thermal_array = [];        % Thermal test array
m_dynamic_array = [];      % Dynamic mass array [kg]
t_dynamic_array = [];      % Dynamic thickness array [mm]
m_mechanical_array = [];   % Mechanical mass array [kg]
t_mechanical_array = [];   % Mechanical thickness array [mm]

for i = 1:row
    %% Dynamic Criterion
    % Condition: f >= fmin = 500 Hz
    % Economic: Smalles m gives lighest plate

    % Minimum thickness based on dynamic criterion [m]
    t_dynamic = (2*pi*fmin)*(R^2) * sqrt(12*(1-(v(i)^2))*p(i)/E(i)) /2.489;

    % Minimum mass based on dynamic criterion [kg]
    m_dynamic = round(p(i)*(pi*(R^2))*t_dynamic,2);

    % Minimum thickness based on dynamic criterion [mm]
    t_dynamic = round(t_dynamic*10^3,2);

    %% Thermal Criterion
    % Condition: Maximum thermal stress <= Fracture stress/factor of safety

    stress_thermal_max = CTE(i) * E(i) * delta_T / (1-v(i));
    if stress_thermal_max <= stress_fracture(i)/N;
        thermal = 1;         % if satisfies
        sprintf("The material passes the thermal criterion");
    else
        thermal = 0;         % if does not satisfy
        sprintf("The material does not pass the thermal criterion");
    end
```

```matlab
    %% Mechanical Criterion
    % Condition: Maximum bending stress <= Fracture stress/factor of safety
    % Economic: Smallest m gives lighest plate

    % Minimum thickness based on mechanical criterion [m]

    t_mechanical = (3/8)*(3+v(i))*p(i)*g*(R^2)*N/stress_fracture(i);

    % Minimum mass based on mechanical criterion [kg]
    m_mechanical = round(p(i)*(pi*(R^2))*t_mechanical,2);

    % Minimum thickness based on mechanical criterion [mm]
    t_mechanical = round(t_mechanical*10^3,2);

    %% Result arrays

    thermal_array = [thermal_array; thermal];        % Thermal test array
    m_dynamic_array = [m_dynamic_array; m_dynamic];  % Dynamic mass array [kg]
    t_dynamic_array = [t_dynamic_array; t_dynamic];  % Dynamic thickness array [mm]

    % Mechanical mass array [kg]
    m_mechanical_array = [m_mechanical_array; m_mechanical];

    % Mechanical thickness array [mm]
    t_mechanical_array = [t_mechanical_array; t_mechanical];

end

%% Results table

Columns = {'Material','Thermal test','Minimum dynamic mass [kg]',...
           'Minimum dynamic thickness [mm]','Minimum mechanical mass...
            [kg]','Minimum mechanical thickness [mm]'};
T = table(mat_name,thermal_array,m_dynamic_array,t_dynamic_array,...
          m_mechanical_array,t_mechanical_array,'VariableNames',Columns);
writetable(T,'Table of results.xlsx');
```